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Transonic Blade-Vortex Interactions: Noise Reduction

Y. Xue* and A. S. Lyrintzis†
University of Minnesota,
Minneapolis, Minnesota 55455

Introduction

AT the present time noise in the design of new or modified helicopters is a matter of great concern. Among the several types of helicopter noise, that due to blade-vortex interactions (BVI) is one of the most important. BVI is the aerodynamic interaction of a rotor blade with the trailing vortex system generated by preceding blades. It usually occurs during helicopter descent, or low-speed maneuvers. It is loud, impulsive in character, and tends to dominate the other sources when it occurs. Also, very complicated BVI patterns arise from tilt-rotor aircraft. Interactions generate the most significant noise when they are intrinsically unsteady, as when the vortex is exactly parallel to the blade, or when the vortex is nearly parallel to the blade. For typical helicopter cases the aerodynamics and aeroacoustics of the interactions are intrinsically transonic. In such cases the flow can be initially modeled by two-dimensional unsteady transonic flow.

Two-dimensional transonic BVI was first studied computationally in the near and midfield by George and Chang[1,2] who used the high-frequency transonic small disturbance equation, including regions of convected vorticity. A comprehensive code, VTRAN2, was then developed.[3–5] The vorticity is bilinearly distributed inside a vortex core and branch cuts are introduced in the x direction. The two-dimensional transonic BVI problem was also solved using the small disturbance theory and the more complex Euler and thin-layer Navier Stokes equations (e.g., Refs. 6–8). A direct comparison of the results obtained from the different methods (from small disturbance to Navier Stokes equations) shows that the results are very similar.[9] At great distances from the airfoil the waves become very difficult to follow because of numerical diffusion and dispersion errors, which led us to use the Kirchhoff method. The Kirchhoff method was introduced[3–5] to extend the numerically calculated nonlinear aerodynamic near-field results to the linear acoustic farfield.

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*Research Assistant, Aerospace Engineering and Mechanics. Student Member AIAA.

†Assistant Professor, Aerospace Engineering and Mechanics. Member AIAA.

In this note several ideas for reduction of transonic BVI noise are introduced and tested. The model used is the two-dimensional high frequency transonic small disturbance equation with regions of distributed vorticity (VTRAN2 code). The far-field noise signals are obtained by using the Kirchhoff method which extends the numerical near-field aerodynamic results to the linear acoustic three-dimensional far field. The results are shown for the airfoil (NACA64A006) shape modification achieved through the addition to various portions of the airfoil lower surface of cosine or NACA 4-digit shapes of different maximum thickness, which demonstrate the idea of shape modification near the leading edge of BVI noise reduction. Also shown are the results for the splitting vortex model, variations of vortex strength, and increasing angle of attack, which are parameters that can be effective to noise reduction. More details can be found in Ref. 10.

Numerical Method (VTRAN2)

VTRAN2 is a code developed for analyzing the interactions of convected regions of vorticity with airfoils using the transonic small disturbance equation. It is based on the ADI implicit scheme of the LTRAN2 code[11] with the inclusion of the high frequency term and the addition of regions of convected vorticity using the cloud-in-cell and multiple branch-cut approach. The vortex can have a free path (convected by the flow) or a prescribed path (miss distance y_v = const, vortex velocity = U_0). The code was modified[5] to include viscosity and monotone switches.

A (213 × 199) nonuniform mesh is used for the calculations. The computational mesh points are clustered more densely near and in front of the airfoil, and are stretched exponentially from the near airfoil region to about 200 chords from the airfoil in the x and 400 in the y direction. More mesh points are added in the y direction for the more accurate evaluation of the normal derivatives on the Kirchhoff surface. The code has a high vectorization level and the CPU time for each two-dimensional case on a Cray-2 computer is about 5 min for 800 time-marching steps.

The numerical simulation method (VTRAN2 code) introduced above is simple and efficient. It is also able to catch the main features of transonic BVI, which are the shock motions around airfoils. The code was shown to agree well with other, more complex approaches, including Euler and thin-layer Navier-Stokes computations.[9] Thus, we expect that our conclusions will also hold if more accurate Euler/Navier-Stokes predictions are used. Three-dimensionality will, of course, influence the results significantly, since the majority of BVIs take place near the tip. Some of the presented results will hold for three-dimensional cases, but only actual three-dimensional calculations can show that.

Kirchhoff's Method for the Far Field

In the past, acoustic analogy has been used for the evaluation of noise signals. This approach starts from the calculation of the nonlinear near and midfield and the far field is found from surface and volume integrals of near and midfield flow and body surfaces. We should note that there are substantial difficulties in including the nonlinear quadrupole term (which requires second derivatives) to the volume integrals, especially around moving shock surfaces. It should also be noted that in the most recent formulation of Farassat et al.[12] the importance of shocks as a potent source of noise is explained. However, only steady shocks are treated. Therefore, in BVI many investigators use data only on the blade surface, which is a less accurate method since shock wave surfaces are not included in the calculation.

Kirchhoff's method includes the calculation of the nonlinear near and midfield with the far-field solutions found from a linear Kirchhoff formulation evaluated on a surface surrounding the nonlinear field. This method provides an adequate matching between the aerodynamic nonlinear near field and the acoustic linear far field. The full nonlinear equations are solved in the first region (near field), usually numerically, and a surface integral of the solution over the control surface gives enough information for the analytical calculation in the second region (far field). The advantage of the method is that nonlinear effects (e.g., shock waves) are accounted for. Also, the surface integrals and the first derivatives needed can be easily evaluated from the near-field computational fluid dynamics (CFD) data; full diffraction and focusing effects are included while eliminating the propagation of the reactive near field.

A Green's function for the linearized governing equation is used to derive a representation for the solution in terms of its values and derivatives on a closed surface S in space. A full three-dimensional formulation is used, because the Green's function is simpler in this case, and because the method can be easily extended to include spanwise variations to model three-dimensional BVI.

Since Kirchhoff's method assumes that linear equations hold outside this control surface S, it must be chosen large enough to include the region of nonlinear behavior. However, due to increasing mesh spacing, the accuracy of the numerical solution is limited to the region immediately surrounding the moving blade. As a result, S cannot be so large as to lose accuracy in the numerical solution for the midfield. Therefore, a judicious choice of S is required for the effectiveness of the Kirchhoff method. A rectangular box-shaped surface is used for the calculations and the VTRAN2 code is used to calculate the solutions on the surface S. Strip theory approximation is used, i.e., the two-dimensional VTRAN2 solution is applied to different segments of the blade in a stripwise manner. For more details the reader is referred to the Refs. 3–5.

Results and Discussion

We use a NACA 64A006 airfoil; the vortex strength was $C_{l_v} = 0.4$ (C_{l_v} is a nondimensional measure of the vortex strength: $C_{l_v} = 2\Gamma/cU_0$) and the vortex miss-distance $y_0 = -0.5$ chords, for a fixed vortex path. The initial vortex position is $x_0 = -9.51$ chords and the freestream velocity is one (arbitrary units) so the vortex passes below the airfoil leading edge at time $T = 9.51$. For the Kirchhoff surface we used a span of 4 chords, x_s (horizontal distance from leading and trailing edge) = 0.25 chords and y_s (vertical distance from airfoil chord) = 1.9–2.5 chords; y_s increases with the Mach number because of the stronger nonlinearities in the larger lateral extent of the flow region; this was as expected from the scaling laws of transonic flow.

The unsteady pressure coefficients have shown (e.g., Ref. 5) the existence of two main disturbances in the transonic region. The first one (I) is believed to be associated with the fluctuating lift coefficient (C_l) and the shock strength variations generated when the vortex passes below the airfoil; disturbance I has a strong forward directivity. The second (II) is believed to be associated with the movement of the shock wave generated at a later time (the vortex is 6–8 chord lengths behind the airfoil) and has a strong downward directivity.

The helicopter transonic BVI shock motions are mainly generated on the lower airfoil surface. Thus, the lower surface of the airfoil needs to be modified, particularly near the leading edge, because disturbance I is generated as the vortex passes below the airfoil leading edge and disturbance II depends on the shock-wave motion which is influenced by changes near the leading edge. Figure 1 shows some results found using the Sikorsky SC1095RN airfoil for a transonic case ($M = 0.8$). (SC1095RN includes modifications of the SC1095 airfoil near the leading edge and was developed in order to reduce BVI noise.) We believe that modifications near the leading edge can both reduce the shock strength and stabilize the shock motion on the lower surface of airfoils. It should be mentioned that other angles of attack were tested (since the helicopter blade always varies angle of attack in flight) with satisfactory results. In subcritical cases however, the noise remains about the same, since it depends mainly on the lift coefficient C_l which does not change.

Figure 2 shows the idea behind the two-vortex model. In order to simplify the problem we have assumed that the distance between the vortices remains constant. However, the total vortex strength still remains the same. This concept can be realized by splitting the trailing vortex using a small airfoil slat in front of the main airfoil. Some exploratory experiments[12] are currently being conducted with such a configuration. Figure 2 shows the effect of splitting the vortex in BVI far-field noise. We also varied the distance between the two vortices from $d = 0.25$ to 0.5 chords. It is shown that the longer the

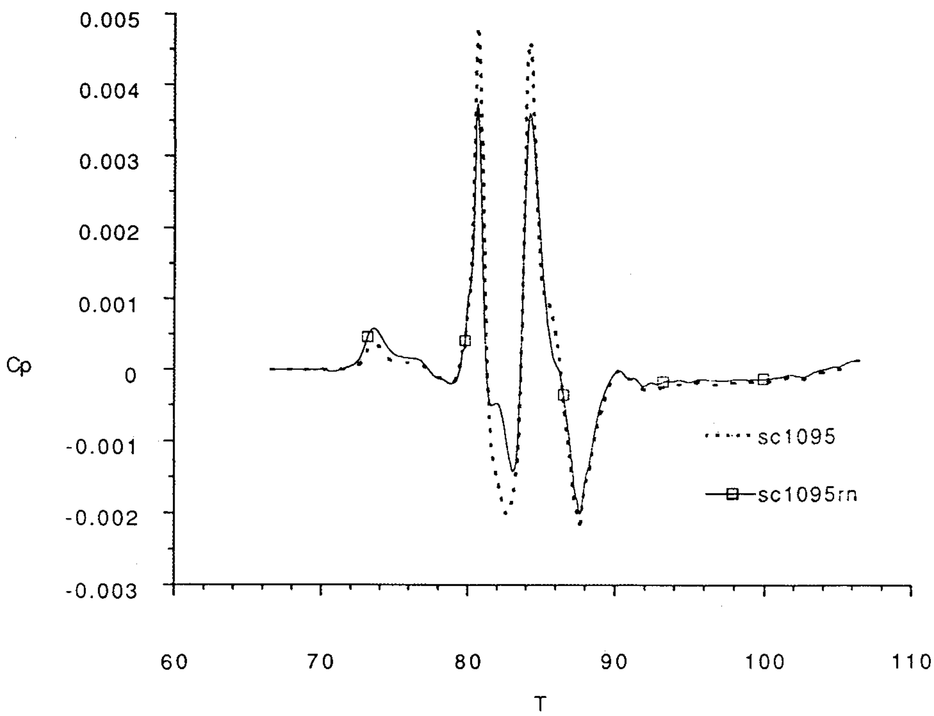

Fig. 1 SC1095 and SC1095RN far-field BVI noise, $M = 0.8$ ($r_v = 30$ chords, $\theta = 30$ deg).

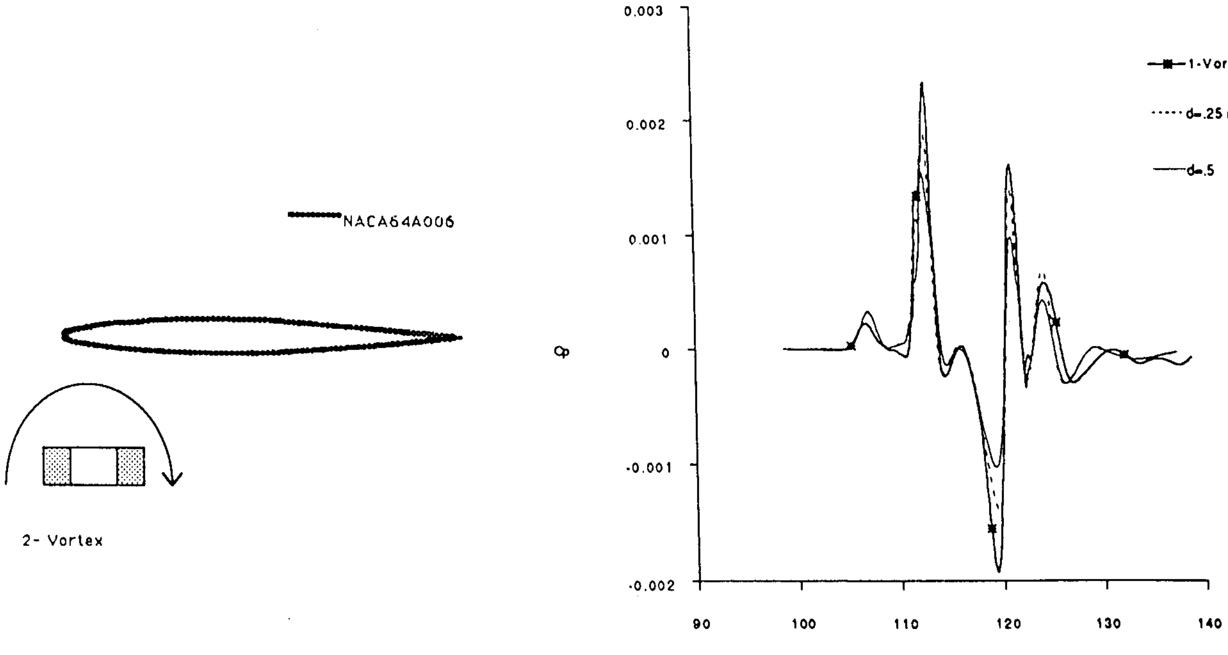

Fig. 2 Vortex splitting model and far-field BVI noise, M = 0.822 (r_v = 50 chords, θ = 30 deg).

distance between the splitting vortices, the more the noise can be reduced.

We should mention here that other parameters can also be effective in reducing BVI noise. The vortex strength is a very important parameter. If we can reduce the vortex strength by one-half by designing a lighter helicopter and adding a trailing-edge flap, for example, then the corresponding far-field noise will be reduced greatly.[10] Yet the vortex miss distance does not have a very strong effect (for distances 0.25–0.5 chords), as shown elsewhere.[4] Also an increase in the angle of attack results in a significant reduction of both disturbances.[10] This explains why the maximum BVI noise always occurs in a certain helicopter descent region. BVI noise can be reduced by a proper combination of advance ratio and descent rate during approach into a noise sensitive area.

Concluding Remarks

An existing numerical finite-difference code VTRAN2 was used to analyze noise due to transonic BVI. The Kirchhoff's method was used to extend the numerically calculated two-dimensional near-field aerodynamic results to the three-dimensional linear acoustic far field. Modified airfoil designs can be very effective in reducing the transonic BVI noise under the same lift and Mach number conditions. The shape of the modification is also very important and further testing is needed to find an optimized shape.

It was shown that splitting the vortex in two can reduce the noise significantly. In fact, both disturbances are reduced. Also, the longer the distance between the two vortices the lower the noise produced. BVI noise will also be greatly reduced by reducing the vortex strength (e.g., by adding a trailing-edge flap) and by increasing the angle of the attack (which explains why thrust can have an effect on BVI noise).

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Eddy Viscosity-Entrainment Correlation

H. Dumitrescu,* S. Ion,† and Al. Dumitrache‡
Institute of Applied Mathematics of the Romanian Academy, Bucharest, Romania

Introduction

THERE are several types of inverse methods to obtain solutions for separated boundary layer. Specifying the distributions of a boundary-layer property (the displacement thickness or the skin friction coefficient) and calculating the corresponding velocity distributions at the outer edge of the boundary layer, these methods eliminate the singularity in the boundary-layer direct methods, at separation.

Inverse approaches for two-dimensional flows using the integral methods have been developed from 1973 onward.[1] In this note, the streamwise momentum integral equation and the entrainment equation[2] are used to describe the pressure-driven separated boundary-layer development in conjunction with the Horton's lag-entrainment equation,[3] and an assumed eddy viscosity-entrainment correlation for equilibrium boundary layers.

Basic Equation

Among the most widely used and successful of many models available for the prediction of turbulent boundary-layer development are versions of the basic entrainment method conceived by Head.[2]

In Head's original scheme, the equations to be integrated are the momentum integral equation

$$\frac{\mathrm{d}}{\mathrm{d}x}(U_e\theta) = \frac{1}{2}C_f U_e - (H+1)\theta\frac{\mathrm{d}U_e}{\mathrm{d}x} \tag{1}$$

(for two-dimensional, constant-density flow) and the entrainment equation which for two-dimensional, constant-density flow can be written in the forms

$$V_E = U_e C_E = \frac{\mathrm{d}}{\mathrm{d}x}\int_0^\delta U\,\mathrm{d}y = \frac{\mathrm{d}}{\mathrm{d}x}[U_e(\delta - \delta^*)]$$
$$= \frac{\mathrm{d}}{\mathrm{d}x}(U_e\theta H_1) \tag{2}$$

In the preceding equations, U_e is the mean velocity at the edge of the boundary layer, C_f is the local skin friction coefficient, U is the time-mean velocity within the boundary layer of thickness δ, so that $U_e = U|_{y=\delta}$ and C_E is the entrainment coefficient. The momentum and displacement thicknesses are denoted by θ and δ^*, respectively, and $H = \delta^*/\theta$, $H_1 = (\delta - \delta^*)/\theta$ are the common shape factor and the mass-flow shape parameter, respectively. The entrainment velocity V_E and the entrainment coefficient C_E are related by $C_E = V_E/U_e$.

The strong history effects characteristic to turbulent boundary layers subject to rapid changes in the streamwise pressure gradient are modeled by Horton's[3] rate equation for C_E, which is actually a simplified form of the stress-transport equation of Bradshaw[4]

$$\frac{\mathrm{d}C_E}{\mathrm{d}x} = \frac{1}{2\delta}(C_{E_{eq}} - C_E) \tag{3}$$

$C_{E_{eq}}$ represents the entrainment coefficient in the equilibrium boundary layers which will be discussed in the following section.

In addition to the basic Eqs. (1), (2), and (3), some auxiliary equations are necessary. These auxiliary equations are based on the Coles' velocity profiles in an extended form.[1] The displacement thickness $\delta^* = \delta^*(x)$ was chosen and then input to the inverse boundary-layer problem. Two ordinary differential Eqs. (1) and (2), and two algebraic equations derived from the velocity profile are available for the solution. This system of coupled differential equations is solved by a fractional step-integration scheme. The uncoupled lag-entrainment Eq. (3) is solved separately.

Review of Existing Correlations

The entrainment coefficient C_E, is defined as the (nondimensional) rate at which fluid from external inviscid flow enters through the outer edge of the boundary layer.

A first attempt to quantify this effect was performed by Head in 1958. In Head's original method,[2] and in Green's later extension of it to compressible flow,[5] C_E was defined empirically as a function of H_1

$$C_E = C_{E_{eq}} = 0.03(H_1 - 3.0)^{-0.617} \tag{4}$$

and respectively

$$C_{E_{eq}} = H_1\{0.0302[(H+1)(H-1)^2/H^3]$$
$$- \tfrac{1}{2}C_f[0.25 + (1.25/H)]\} \tag{5}$$

In 1967, based on a reduced form of the turbulent energy equation, "diffusion" = "advection," Bradshaw[4] established the relationship

$$C_E = 10(\tau_{max}/\rho U_e^2)^{1.0} \tag{6}$$

where τ_{max} designates the maximum shear stress. This correlation was assumed to be held both in equilibrium and nonequilibrium boundary layers, and even in the mixing layer.

In 1969, Horton, in a lag-entrainment method,[3] suggested that $C_{E_{eq}}$ can be directly related to the mean shear stress of the outer part of the boundary layer by the relationship

$$C_{E_{eq}} = 1.585(\tau_{1/2}/\rho U_e^2)^{0.69} \tag{7}$$

where $\tau_{1/2}$ is the shear stress at $y/\delta = 0.5$. It is computed with the Cebeci-Smith turbulence model[6] for the outer layer which is highly inaccurate for equilibrium layers in strong adverse pressure gradient.

Head and Galbraith[7] use a different approach in determining $C_{E_{eq}}$. They started by using the general entrainment definition, as the rate at which fluid is crossing a line of constant U/U_e, and at the first boundary layer this becomes

$$C_E = \frac{1}{U_e}\lim_{y\to\infty}\left(\frac{\frac{1}{\rho}\frac{\partial\tau}{\partial y}}{\frac{\partial U}{\partial y}}\right) \tag{8}$$

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*Dr. Engineering, Senior Research Scientist, Fluid Dynamics Department, P.O. Box 76-177, 79622.
†Mathematician, Research Scientist, Fluid Dynamics Department, P.O. Box 76-177, 79622.
‡Aerospace Engineer, Research Scientist, Fluid Dynamics Department, P.O. Box 76-177, 79622.